RENORMALIZATION OF THE $1/N$ EXPANSION AND CRITICAL BEHAVIOUR OF (2 + 1)-DIMENSIONAL SUPERSYMMETRIC NON-LINEAR SIGMA-MODELS

E.R. NISSIMOV and S.J. PACHEVA
Institute of Nuclear Research and Nuclear Energy, Sofia 1184, Bulgaria

ABSTRACT. Renormalized $1/N$ expansion in both high- and low-temperature phases as well as of the critical theory of three-dimensional supersymmetric generalized non-linear sigma-models is constructed and scaling laws for the Green's functions near the critical point with only two independent critical exponents are established.

1. In a recent note [1] (hereafter referred to as I) quantum three-dimensional ($D = 3$) supersymmetric (SS) generalized non-linear sigma-models (GNLSM's) only real (complex) Grassmannians $O(N)/O(N-n) \times O(n)$ $(U(N)/U(N-n) \times U(n))$ were considered in the framework of the non-perturbative $1/N$ expansion. The following results were found:

A second-order phase transition occurs at a certain critical value T_c of the coupling constant (temperature) T. The corresponding high-temperature phase (HTP) is SS, $O(N)$ $(U(N))$ isotopic- and $O(n)(U(n))$ gauge-symmetric with the particle spectrum consisting of $n \cdot N$ boson-fermion pairs of equal (dynamically generated) mass and of $\frac{1}{2}n(n-1)\,(n^2)$ massless transverse 'gluons'. The low-temperature Higgs–Goldstone phase (LTP) is SS with spontaneous breaking of the internal $O(N) \times O(n)_{\text{gauge}}(U(N) \times U(n)_{\text{gauge}})$ symmetry and its particle spectrum consists of only $n(N-1)$ pairs of massless Goldstone bosons and fermions, all other particles occurring in HTP being here 'confined'.

In the present note we consider the renormalization of the $1/N$ expansion in both HTP and LTP as well as in the critical theory (CT), prove its renormalizability (standard dimensional and symmetry arguments turn out not to be sufficient for the latter, see the Lemma below), and derive scaling laws for the Green's functions near T_c with only two independent critical exponents expressed through the anomalous dimensions of the Φ and Σ-superfields.

The superspace Lagrangian density describing $D = 3$ SS GNLSM's (we consider for definiteness the complex case) reads (cf. (I. 1–3)):

$$\begin{aligned}
\mathcal{L}(x, \theta)) &= \tfrac{1}{2}(\overline{\nabla}_\alpha \Phi^*)_a^k (\nabla_\alpha \Phi)_a^k - \Phi_a^{*k} \underset{\sim}{\Sigma}^{kl} \Phi_a^l + Nna_0 \Sigma_0 - \\
&\quad - N \operatorname{tr}(\underset{\sim}{\mathscr{B}} \overline{\mathscr{D}}_\alpha \underset{\sim}{\mathscr{A}}_\alpha) + \operatorname{tr}(\underset{\sim}{\chi}^* \overline{\mathscr{D}}_\alpha \nabla_\alpha \underset{\sim}{\chi}); \\
(\nabla_\alpha \Phi)_a^k &= \mathscr{D}_\alpha \Phi_a^k + i \underset{\sim}{\mathscr{A}}_\alpha^{kl} \Phi_a^l, \qquad (\nabla_\alpha \underset{\sim}{\chi})^{kl} = \mathscr{D}_\alpha \underset{\sim}{\chi}^{kl} + i [\underset{\sim}{\mathscr{A}}_\alpha, \underset{\sim}{\chi}]^{kl}; \qquad (1) \\
\mathscr{D}_\alpha &= \partial/\partial \overline{\theta}_\alpha - i(\not{\partial}\theta)_\alpha, \quad \not{\partial} = \gamma^\mu \partial_\mu; \\
\underset{\sim}{X}^{kl} &= X_A \lambda_A^{kl} (\underset{\sim}{X} = \underset{\sim}{\mathscr{A}}_\alpha, \underset{\sim}{\mathscr{B}}, \underset{\sim}{\Sigma}, \underset{\sim}{\chi} \text{ etc.}), \quad \operatorname{tr} \lambda_A \lambda_B = n\delta_{AB}, \lambda_A^2 = \mathbb{1};
\end{aligned}$$

Letters in Mathematical Physics **5** (1981) 333–340. 0377–9017/81/0054–0333 $00.80.

$a_0 \equiv \mu/T$, μ being an arbitrary mass scale parameter. Here the auxiliary superfield $\underset{\sim}{\mathscr{B}}$ enforces the Landau gauge $\bar{\mathscr{D}}_\alpha \mathscr{A}_\alpha = 0$; $\underset{\sim}{\chi}$ are the corresponding Faddeev–Popov superfield ghosts. The hermitian $n \times n$ matrices λ_A, $A = 0, 1, ..., n^2 - 1$, $\lambda_0 = \mathbb{1}$, span a (hermitian) basis of U(n)-Lie algebra. As in I, the summation over repeated indices ('isotopic' ones $a = 1, ..., N$, 'color' ones $k, l = 1, ..., n$, adjoint-U(n) ones A, B, Lorentz-spinor ones $\alpha = 1, 2$) is understood and the latter will often be suppressed for brevity. In components (cf. (I. 2)) the model (1) looks as:

$$\mathcal{L}(x) = (\nabla_\mu \varphi)^*(\nabla^\mu \varphi) + \frac{i}{2}\bar{\psi}\overleftrightarrow{\nabla}\psi + (4Nna_0)^{-1}\sum_A (\bar{\psi}\lambda_A\psi)^2 - \varphi^*\underset{\sim}{\alpha}\varphi +$$

$$+ Nna_0\alpha_0 + \bar{\psi}\underset{\sim}{\kappa}\varphi + \varphi^*\underset{\sim}{\kappa}\psi - N\,\mathrm{tr}(\underset{\sim}{B}\partial_\mu \underset{\sim}{A}^\mu) + \mathrm{tr}\,(\underset{\sim}{\eta}^*\partial^\mu \nabla_\mu \underset{\sim}{\eta}); \qquad (2)$$

$$\nabla_\mu \varphi = \partial_\mu \varphi + i\underset{\sim}{A}_\mu \varphi, \quad A_{\mu,B} = (2Nna_0)^{-1}\{i\varphi^*\lambda_B \overleftrightarrow{\partial}_\mu \varphi + \bar{\psi}\lambda_B\gamma_\mu\psi\},$$

$\underset{\sim}{B}$ and $\underset{\sim}{\eta}$ being the usual Landau and Faddeev–Popov ghost-fields, and the corresponding SS transformations are (cf. [2]):

$$\delta\varphi = \bar{\epsilon}\psi, \qquad \delta\psi = -i(\partial\!\!\!/\varphi)\epsilon + \epsilon(2Nna_0)^{-1}(\bar{\psi}\lambda_A\psi)\lambda_A\varphi.$$

The auxiliary fields $\underset{\sim}{\alpha}$ and $\underset{\sim}{\kappa}$ in (2) enforce the non-linear constraints:

$$\varphi^*\lambda_A\varphi - Nna_0\delta_{A0} = 0, \qquad \bar{\psi}\lambda_A\varphi = \varphi^*\lambda_A\psi = 0.$$

2. Now we are going to formulate a unified renormalization procedure for the $1/N$ expansion of (1) which is an appropriate modification of the Zimmermann–Lowenstein 'soft' mass renormalization scheme [3]. It is well known that the latter is especially well suited for treatment of theories with massless particles (for application to the non-SS GNLSM's, see [4, 5]). The main point is to make dimensional parameters (in the present context – the mass m of Φ, its large N vacuum expectation value (VEV) $\hat{\varphi}$ and the θ's) variable in such a way that the subtraction operators in the Zimmermann 'forest' formula [6], or equivalently, in the Bogoliubov's R-operation [7] (Equation (4) below) act also on them. This will assure:

(1) minimal ultraviolet (UV) subtractions,
(2) no generation of artificial infrared (IR) divergences through UV subtractions at zero external momenta,
(3) fulfilment of Ward identities for the spontaneously broken internal U(N) $\times$ U(n)$_{\text{gauge}}$ symmetry in LTP (see Equations (9) and (10) below).

For (1) we are led to the following 'soft' modification of the $1/N$ supergraph propagators (cf. (I. 12–15)):

$$\langle \Phi_a^k \Phi_b^{*l}\rangle^{(0)} = -i\delta_{ab}\delta^{kl}\{[sm + (1-s)\mu]^2 - p^2\}^{-1} e^{\bar{\theta}p\!\!\!/\theta'}(1 + sm\delta(\theta - \theta')), \qquad (3a)$$

$$\langle \Sigma_A \Sigma_B\rangle^{(0)} = \begin{cases} -i(Nn)^{-1}\delta_{AB}[(4m^2 - p^2)F(p^2)]^{-1} e^{\bar{\theta}p\!\!\!/\theta'}(1 - 2sm\delta(\theta - \theta')) \\ -8i(Nn)^{-1}\delta_{AB}[(-p^2)^{1/2} + 8s|\hat{\varphi}|^2]^{-1} e^{\bar{\theta}p\!\!\!/\theta'}, \end{cases} \qquad (3b)$$

$$\langle \mathscr{A}_A \overline{\mathscr{A}}_B \rangle^{(0)} = \begin{cases} 2i(Nn)^{-1}\delta_{AB}[p^2(4m^2-p^2)F(p^2)]^{-1}\not{p}(2sm+\not{p})\Pi(p;\theta,\theta') \\ 16i(Nn)^{-1}\delta_{AB}[(-p^2)^{1/2}+16s|\hat{\varphi}|^2]^{-1}\Pi(p;\theta,\theta'); \end{cases} \tag{3c}$$

$$F(p^2) = (8\pi)^{-1}\int_0^1 \mathrm{d}x\,[m^2-p^2x(1-x)]^{-1/2}; \qquad \Pi(p;\theta,\theta') = \tfrac{1}{2}\,\mathrm{e}^{\bar{\theta}\not{p}\theta'}(\delta(\theta-\theta')-1/\not{p}),$$

$$\hat{\varphi}^*\lambda_A\hat{\varphi} \equiv \delta_{A0}n|\hat{\varphi}|^2, \quad \not{p} = \gamma^\mu p_\mu, \; s\in[0,1].$$

In all LTP-vertices containing $\hat{\varphi}$ ($-i\hat{\varphi}^*\underset{\sim}{\Sigma}\phi$, $(i/2)\hat{\varphi}^*\underset{\sim}{\overline{\mathscr{A}}}_\alpha\underset{\sim}{\mathscr{A}}_\alpha\Phi$ etc., cf. (I.11)) we substitute $\hat{\varphi}\to s^{1/2}\hat{\varphi}$ [4, 5].

Renormalization of $1/N$ (connected) supergraphs G is achieved by means of the R-operation recurrence formula [7]:

$$R_G(p^G, k^G;\theta^G) = (1-\tau^{\delta(G)})\overline{R}_G(p^G, k^G;\theta^G), \tag{4a}$$

$$\overline{R}_G(p^G, k^G;\theta^G) = \sum_{\{\Gamma\}}{}' \int \Pi\,\mathrm{d}^2\varphi^{G/\{\Gamma\}} I_{G/\{\Gamma\}}(p^{G/\{\Gamma\}}, k^{G/\{\Gamma\}}, \theta^{G/\{\Gamma\}}|\varphi^{G/\{\Gamma\}}) \times$$

$$\times \prod_{\Gamma_j\in\{\Gamma\}} [-\tau^{\delta(\Gamma_j)}\overline{R}_{\Gamma_j}(p^{\Gamma_j}, k^{\Gamma_j};\theta^{\Gamma_j},\varphi^{\Gamma_j})]; \tag{4b}$$

$$\theta^{G/\{\Gamma\}}, \qquad \theta^{\Gamma_j}\subset\theta^G, \qquad \theta^{\Gamma_j}\subset\varphi^{G/\{\Gamma\}}.$$

In Equation (4) all notations are standard: $I_\Gamma(p^\Gamma, k^\Gamma;\theta^\Gamma|\varphi^\Gamma)$ is the unrenormalized integrand, $R_\Gamma(p^\Gamma, k^\Gamma;\theta^\Gamma)$ – the renormalized integrand (integrated over φ^Γ) of a supergraph Γ; $p^\Gamma, k^\Gamma, \theta^\Gamma, \varphi^\Gamma$ are sets of external and internal (loop) momenta, external and internal Grassmann factors respectively; the sum in (4b) runs over all partitions $\{\Gamma\}=\emptyset$, $\{\Gamma_1, ..., \Gamma_c\}$, $\{\Gamma\}\neq G$, of G into mutually disjoint connected subgraphs $\Gamma_j\subset G$; $G/\{\Gamma\}$ denotes the corresponding reduced graph. Subtraction operators τ^ω are defined by the properties (cf. [8]):

$$\tau^\omega f(p,s) = t^\omega_{p,s}f(p,s); \qquad \tau^\omega\theta_\alpha f(p,s;\theta) = \theta_\alpha\tau^{\omega+1/2}f(p,s;\theta),$$

where $f(p,s)$, $f(p,s;\theta)$ are arbitrary functions and $t^\omega_{p,s}$ is the standard Taylor subtraction operator of order ω in the variables p (external momenta in (4)), s. By definition $\tau^{\delta(\Gamma)}0$ for Γ a one-particle reducible (sub)graph. After implementing all subtractions in (4) one sets $s = 1$. Canonical UV degree (dimension) $\delta(\Gamma)$ of a supergraph Γ is computed to be ($\dim\theta_\alpha = -\frac{1}{2}$, $\dim\hat{\varphi} = \frac{1}{2}$ etc.):

$$\delta(\Gamma) = 3 - \tfrac{1}{2}(L_\Phi(\Gamma)+L_{\mathscr{A}}(\Gamma)) - L_\Sigma(\Gamma) - V_e(\Gamma)$$

where $L_{\Phi,\Sigma,\mathscr{A}}(\Gamma)$ and $V_e(\Gamma)$ are numbers of external Φ_-, Σ_-, $\mathscr{A}_-$-lines and external vertices of Γ respectively.

The following general representations are due to SS:

$$\int \Pi\, d^2\varphi I_\Gamma(p^\Gamma, k^\Gamma; \theta^\Gamma | \varphi^\Gamma) = \exp\left\{\sum_{r=1}^{V_e(\Gamma)-1} \bar\theta_{V_e} \not p_r \theta_r\right\} \times$$

$$\times \sum_{h=0(1)}^{V_e(\Gamma)-1} P_\Gamma^{[2h]}((\theta_r^\Gamma - \theta_{V_e}^\Gamma)) \times J_\Gamma^{[2h]}(p^\Gamma, k^\Gamma);$$

$[2h] = 2h$ (I_G even), $2h - 1$ (I_G odd).

Here $p_\Gamma^{[2h]}$ denote polynomials in all differences $\theta_r^\Gamma - \theta_{V_e}^\Gamma$, homogeneous of degree $[2h]$; I_G even (odd) means I_G an even (odd) element of a Grassmann algebra. From Equations (4b) and (5) we have:

$$\tau^{\delta(\Gamma)} \bar R_\Gamma(p^\Gamma, k^\Gamma, \theta^\Gamma) = \exp\left\{\sum_{r=1}^{V_e(\Gamma)-1} \bar\theta_{V_e} \not p_r \theta_r\right\} \sum_{h=0(1)}^{V_e(\Gamma)-1} \bar p_\Gamma^{[2h]}((\theta_r^\Gamma - \theta_{V_e}^\Gamma)) \times$$

$$\times\, t_{p^\Gamma, s}^{\omega_{[2h]}(\Gamma)} \bar J_\Gamma^{[2h]}(p^\Gamma, k^\Gamma); \tag{6}$$

$$\omega_{[2h]}(\Gamma) = \delta(\Gamma) + [2h] \leqslant \omega_{\max}(\Gamma) \equiv \omega_{[2(V_e - 1)]}(\Gamma),$$

$$\omega_{\max}(\Gamma) = 2 - \tfrac{1}{2}(L_\Phi(\Gamma) + L_{\mathscr{A}}(\Gamma)) - L_\Sigma(\Gamma). \tag{7}$$

Equation (6) shows that the R-operation (4) does not spoil the SS since $\tau^{\delta(\Gamma)}$ does not affect the factor $\exp\{\Sigma \bar\theta_{V_e} \not p_r \theta_r\}$. Note that, according to (3a), UV subtractions (4) in LTP and CT are made at non-zero Φ-mass, thus avoiding IR divergences.

3. It follows from (4), (6) and (7) that the form of the (finite) counterterms of the renormalized theory (1) is dictated by $\omega_{\max}(\Gamma)$ (7) (i.e., for which values of $L_{\Phi, \Sigma, \mathscr{A}}(\Gamma)$ $\omega_{\max}(\Gamma) \geqslant 0$).

LEMMA. *The following equalities are valid*:

$$\bar J_\Gamma^{[2(V_e(\Gamma)-1)]}(p^\Gamma, k^\Gamma) = \begin{cases} sm\, \bar J'_\Gamma(p^\Gamma, k^\Gamma) & \text{(HTP)} \\ 0 & \text{(LTP, CT)} \end{cases}$$

whenever $(L_\Phi(\Gamma), L_\Sigma(\Gamma), L_{\mathscr{A}}(\Gamma)) = (0, 1, 0), (0, 2, 0), (L, 0, 0), L = 1, \ldots, 4$, *with* $\bar J'_\Gamma(p^\Gamma, k^\Gamma)$ *having UV degree* $\omega_{\max}(\Gamma) - 1$.

Here we shall skip the simple proof of this Lemma. The latter, together with (6) and (7), tells us that:

(a) All UV divergences are logarithmic;

(b) counterterms $\underset{\sim}{\Sigma}^2(x, \theta)$, $(\Phi^*\Phi)^2(x, \theta)$ are absent.

Absence of $\underset{\sim}{\Sigma}^2$-counterterm plays a crucial role for the renormalizability of (1). Namely, it guarantees the validity of the quantum equations of motion for $\underset{\sim}{\Sigma}$:

$$\langle \mathcal{N}[\Phi'^*\lambda_A\Phi' P](x,\theta)\rangle^{J,L,K} = Nna\delta_{A0}\langle \mathcal{N}[P](x,\theta)\rangle^{J,L,K};$$

$$\Phi' \equiv \Phi + s^{1/2}N^{1/2}\hat{\varphi}, \quad a \equiv a_0 + m\widetilde{a},$$

$$\langle \mathcal{F}\rangle^{J,K,L} \equiv \mathcal{R}\{Z^{-1}[J,L,K]\int \Pi\, \mathrm{d}\Phi\, \mathrm{d}\Phi^*\, \mathrm{d}\underset{\sim}{\Sigma}\, \mathrm{d}\underset{\sim}{\mathcal{A}}\, \mathrm{d}\underset{\sim}{\mathcal{B}}\, \mathrm{d}\underset{\sim}{\chi}\, \mathrm{d}\underset{\sim}{\chi}^*\mathcal{F} \times$$

$$\times \exp i\int \mathrm{d}^3x\, \mathrm{d}^2\theta\, [\mathcal{L}(x,\theta) + \mathcal{L}_{\text{c.t.}}(x,\theta) + J^*\Phi + \Phi^*J + L_A\Sigma_A + \bar{K}_A\mathcal{A}_A]\}; \tag{8}$$

which are proper quantum analogues of the nonlinearity constraints: $\Phi^*\lambda_A\Phi - Nna_0\delta_{A0} = 0$ (see (1)). In Equation (8) P denotes an arbitrary composite superfield; $\widetilde{a}$ is the coefficient of the finite counterterm $m\widetilde{a}\Sigma_0$ in HTP (in LTP and CT $\widetilde{a} = 0$); $Z[J, L, K]$ is the quantum generating functional; $\mathcal{R}$ denotes the graph by graph R-operation (4); $\mathcal{L}_{\text{c.t.}}$ represents the sum of all admissiable counterterms. Symbols $\mathcal{N}$ in (8) denote canonical Zimmermann normal products of the corresponding composite fields. Supergraphs with normal product vertex insertions are renormalized by means of (4) exactly in the same manner as in the non-SS case [6]. Equation (8) can be proved in complete analogy with the non-SS case [5] (unless in the latter case here no additional terms in the right-hand side of (8) arise due to the lower UV degrees (7)).

Further ingredients in proving the renormalizability of (1) are:

(i) Ward identities for the (spontaneously broken in LTP) U(n) gauge invariance (SS Slavnov–Taylor-type identities):

$$n\bar{\mathcal{D}}_\alpha\langle \mathcal{N}[[\underset{\sim}{\chi}^*, \nabla_\alpha\underset{\sim}{\chi}]_A - N(\nabla_\alpha\underset{\sim}{\mathcal{B}})_A](x,\theta)\rangle^{J,L,K} = \langle \Phi'^*\lambda_A J -$$

$$- J^*\lambda_A\Phi' + [\underset{\sim}{L}, \underset{\sim}{\Sigma}]_A - n(\nabla_\alpha\underset{\sim}{\bar{K}}_\alpha)_A - i\delta S^{\Omega}_{\text{c.t.}}/\delta\omega_A(x,\theta)|_{\omega=0}\rangle^{J,L,K}; \tag{9}$$

$$S^{\Omega}_{\text{c.t.}} \equiv \int \mathrm{d}^3x\, \mathrm{d}^2\theta\, \mathcal{L}_{\text{c.t.}}(\Phi'^{\Omega}, \underset{\sim}{\Sigma}^{\Omega}, \underset{\sim}{\mathcal{A}}^{\Omega}, \underset{\sim}{\chi}^{\Omega}), \qquad \Omega(x,\theta) = \exp\{i\lambda_A\omega_A(x,\theta)\} \in \mathrm{U}(n),$$

$$\Phi'^{\Omega} (= (\Phi + s^{1/2}N^{1/2}\hat{\varphi})^{\Omega}) = \Omega^{-1}\Phi', \quad \underset{\sim}{\Sigma}^{\Omega} = \Omega^{-1}\underset{\sim}{\Sigma}\Omega, \underset{\sim}{\chi}^{\Omega} = \Omega^{-1}\underset{\sim}{\chi}\Omega,$$

$$\underset{\sim}{\mathcal{A}}^{\Omega}_\alpha = \Omega^{-1}\underset{\sim}{\mathcal{A}}_\alpha\Omega + \Omega^{-1}\mathcal{D}_\alpha\Omega.$$

(ii) Ward identities for the (spontaneously broken in LTP) U(N) isotopic symmetry (with $\lambda^{(M)}$, $M = 0, 1, \ldots, N^2 - 1$, being a generator basis):

$$\tfrac{1}{2}\bar{\mathcal{D}}_\alpha\langle \mathcal{N}[\Phi'^*\lambda^{(M)}\nabla_\alpha\Phi'](x,\theta)\rangle^{J,L,K} = \langle \Phi'^*\lambda^{(M)}J - J^*\lambda^{(M)}\Phi' +$$

$$+ \Phi'^*\lambda^{(M)}\frac{\delta S_{\text{c.t.}}}{\delta\Phi^*(x,\theta)} - \frac{\delta S_{\text{c.t.}}}{\delta\Phi(x,\theta)}\lambda^{(M)}\Phi'\rangle^{J,L,K}. \tag{10}$$

Equations (9), (10) are simple consequences of the renormalized according to (4) quantum equations of motion for Φ, $\mathcal{A}$ and χ. Due to the occurrence of the auxiliary superfield $\underset{\sim}{\mathcal{B}}$ on the left-hand side of (9) one must use the mixed $1/N$ propagator (the only way $\underset{\sim}{\mathcal{B}}$ can couple to other elements in a $1/N$ supergraph):

$$\langle \mathscr{B}_A \mathscr{A}_{\alpha, B} \rangle = -i(2Nn)^{-1} \delta_{AB} (p^2)^{-1} \, e^{\bar{\theta} \not{p} \theta'} \not{p} (\theta - \theta')_\alpha .$$

Identities (8), (9) and (10) restrict the list of admissible counterterms to the following ones:

$$(\bar{\nabla}_\alpha \Phi'^*)(\nabla_\alpha \Phi'), \qquad \Sigma_0 , \qquad \mathrm{tr}\,(\underset{\sim}{\chi}^* \bar{\mathscr{D}}_\alpha \nabla_\alpha \underset{\sim}{\chi})$$

(the last one is irrelevant since external χ-sources are absent).

Thus, we arrive at the following renormalized (effective in the Zimmermann sense [6]) Lagrangian:

$$\mathcal{L}_{\mathrm{eff}}(x, \theta) = \mathcal{L}(x, \theta) + \mathcal{L}_{\mathrm{c.t.}}(x, \theta) = \mathcal{N}[\tfrac{1}{2}(1 + b)(\bar{\nabla}_\alpha \Phi'^*)(\nabla_\alpha \Phi') - m \Phi'^* \Phi' -$$

$$- \Phi'^* \underset{\sim}{\Sigma} \Phi' + Nna\Sigma_0 - N\,\mathrm{tr}\,(\underset{\sim}{\mathscr{B}} \bar{\mathscr{D}}_\alpha \underset{\sim}{\mathscr{A}}_\alpha) + \mathrm{tr}\,(\underset{\sim}{\chi}^* \bar{\mathscr{D}}_\alpha \nabla_\alpha \underset{\sim}{\chi})](x, \theta). \tag{11}$$

The coefficients a, b are determined by the physical normalization conditions:

(a) $N^{1/2} \hat{\varphi} = \langle \Phi(x, \theta) \rangle^{J=L=K=0}$

(b) The full two-point function:

$$\langle \Phi_a^k \Phi_b^{*l} \rangle = -i\delta_{ab} \delta^{kl} \, e^{\bar{\theta} \not{p} \theta'} [G_1(-p^2/\mu^2) + mG_2(-p^2/\mu^2)\, \delta(\theta - \theta')] \times$$

$$\times \, [m^2 G_2^2(-p^2/\mu^2) - p^2 G_1^2(-p^2/\mu^2)]^{-1} \tag{12}$$

has a simple pole at $p^2 = m^2$ (HTP), i.e., the parameters $m, \hat{\varphi}$ entering the $1/N$ graphical elements (3) are the exact mass and the exact VEV of Φ respectively.

(c) Normalization of Φ:

$$\langle \Phi_a^k \Phi_b^{*l} \rangle_{(\perp)} |_{p^2 = -\mu^2} = \langle \Phi_a^k \Phi_b^{*l} \rangle_{(\perp)}^{(0)} |_{p^2 = -\mu^2} \tag{13}$$

where '$\perp$' denotes projection of Φ orthogonal to $\hat{\varphi}$ in LTP.

Conditions (12) and (13) give:

$$G_1(-m^2/\mu^2) = G_2(-m^2/\mu^2), \qquad G_1(1) = G_2(1) = 1.$$

4. To describe scaling behaviour of Green's functions near T_c one has to introduce temperature- and 'magnetic-field'-perturbation (TMFP) on the CT:

$$\mathcal{L}_{\mathrm{TMFP}}(x, \theta) = \mathcal{L}_{\mathrm{eff}} |_{m=0, \hat{\varphi}=0} - Nn\mu t \Sigma_0(x, \theta) + H^* \Phi(x, \theta) + \Phi^*(x, \theta) H \tag{14}$$

(with t describing small deviations of T away from T_c and H being a supersymmetric constant source $H(x, \theta) = H$). Naive expansion in powers of TMFP would create IR divergences because of the UV super-renormalizability of TMFP. However, in complete analogy with the non-SS case [4], we can perform a partial resummation of TMFP (14) after which the new $1/N$ expansion is defined by the Lagrangian:

$$\mathcal{L}'_{\mathrm{TMFP}}(x,\theta) = \mathcal{L}_{\mathrm{eff}}\big|_{m \rightsquigarrow 4\pi(\mu t + |\hat{\varphi}|^2),\, \hat{\varphi} \rightsquigarrow \hat{\varphi},\, a \rightsquigarrow \hat{a},\, b \rightsquigarrow \hat{b}}\,. \tag{14'}$$

In Equation (14′) '$\rightsquigarrow$' means 'replaced by'; $N^{1/2}\underline{\hat{\varphi}} = \langle\Phi\rangle_{t,H}^{J=L=K=0}$ is the exact VEV in the presence of TMFP; $\underline{\hat{\varphi}}$ is treated as an independent parameter instead of H. The new counterterm coefficients $\hat{a}$, $\hat{b}$ are IR finite and are given explicitly as functions of the old ones a, b (11) (in the $1/N$ expansion), cf. [4].

Now, by means of a SS extension of the well-known method of differential-vertex operations [9], accounting for the explicit form of the present $1/N$ renormalization scheme (4), (3), we can derive the following renormalization group equations for the TMFP theory (14′) (c.f. [4, 5] for the non-SS GNLSM's):

$$\left\{\mu\frac{\partial}{\partial\mu} - (1-\zeta_\Sigma)t\frac{\partial}{\partial t} + \zeta_\Phi\left(J\frac{\delta}{\delta J} + J^*\frac{\delta}{\delta J^*} - \hat{\varphi}\frac{\partial}{\partial\hat{\varphi}} - \hat{\varphi}^*\frac{\partial}{\partial\hat{\varphi}^*}\right) + \right.$$

$$\left. + \zeta_\Sigma L\frac{\delta}{\delta L}\right\} W[J, L, K; t, \underline{\hat{\varphi}}] = 0, \quad d_\Phi = \tfrac{1}{2} + \zeta_\Phi,\ d_\Sigma = 1 + \zeta_\Sigma, \tag{15}$$

where $iW[\ldots] = \log Z[\ldots]$ is the generating functional of connected Green's functions of (14′) and $d_{\Phi,\Sigma}$ are the full anomalous scale dimensions of Φ, Σ. Solution of Equations (15) by means of the method of characteristics gives a SS generalization of Kadanoff's scaling laws (for a review of the non-SS case, see, e.g., [10]):

$$W[J^{(\kappa)}, L^{(\kappa)}, K^{(\kappa)}; \kappa^{1-\zeta_\Sigma}t, \kappa^{1/2+\zeta_\Phi}\underline{\hat{\varphi}}] = W[J, L, K; t, \underline{\hat{\varphi}}],$$

$$J^{(\kappa)}_{(x,\theta)} = \kappa^{3/2-\zeta_\Phi}J(\kappa x, \kappa^{1/2}\theta), \qquad L^{(\kappa)}_{(x,\theta)} = \kappa^{1-\zeta_\Sigma}L(\kappa x, \kappa^{1/2}\theta), \tag{16}$$

$$K^{(\kappa)}(x,\theta) = \kappa^{3/2}K(\kappa x, \kappa^{1/2}\theta),$$

κ being an arbitrary scaling parameter, with only two independent critical exponents η, ν (for standard definitions, see, e.g., [10]):

$$\eta = 2\zeta_\Phi, \qquad \nu = (1-\zeta_\Sigma)^{-1}.$$

In the leading $1/N$ order we find: $\eta^{(1)} = -4n/N\pi^2$.

In a subsequent work, it will be shown that the model (1) (or (2)) at $T = T_c$ is a universal (in Kadanoff's sense) SS, gauge- and scale-invariant critical theory of a class of $1/N$ expandable SS and *non-SS* Higgs models with fermions which are non-renormalizable in the naive perturbation theory.

REFERENCES

1. Nissimov, E.R. and Pacheva, S.J., *Lett. Math. Phys.* **5**, 67 (1981).
2. D'Adda, A., Lüscher, M., and Di Vecchia, P., *Nucl. Phys.* **B152**, 125 (1979).
3. Lowenstein, J. and Zimmermann, W., *Comm. Math. Phys.* **46**, 105 (1976); Lowenstein, J., *Comm. Math. Phys.* **47**, 53 (1976).
4. Nissimov, E.R., and Pacheva, S.J., *Theor. Math. Phys.* **41**, 55, 220 (1979); *Bulg. J. Phys.* **6**, 610 (1979); 7, 16 (1980).
5. Aref'eva, I.Ya., Nissimov, E.R., and Pacheva, S.J., *Comm. Math. Phys.* **71**, 213 (1980).
6. Zimmermann, W., *Ann. Phys.* **77**, 536 (1973).
7. Bogoliubov, N.N. and Shirkov, D.V., *Introduction to the Theory of Quantized Fields,* 3rd. edn., Nauka, Moscow (1976).
8. Piguet, O. and Rouet, A., *Nucl. Phys.* **B108**, 265 (1976).
9. Lowenstein, J., *Comm. Math. Phys.* **24**, 1 (1971).
10. Brezin, E., Le Guillou, J., and Zinn-Justin, J., in C. Domb & M. Green (eds.) *Phase Transitions and Critical Phenomena*, vol. 6, Academic Press, New York, 1976.

(Received February 25, 1981)